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The Heliographic Coordinates of Sun-spots and Faculæ on the Stonyhurst Drawings. By the Rev. A. L. Cortie, S.J.

The photographic method of registering the positions of sun-spots and faculæ, and their micrometrical measurement from the photographs as practised at Greenwich, is doubtless the most perfect of those in use. Taking the positions as reduced, and published in the annual volumes of "Spectroscopic and Photographic Results" as standard, it will be useful to compare with them some positions which have been obtained from the Stonyhurst drawings by the use of a set of orthographic projections of the parallels of latitude and meridians of longitude, and to explain the method by which these positions have been obtained. For the process of measurement by means of projections is most simple and expeditious, and, moreover, is susceptible of considerable accuracy. The drawings themselves are made by projecting the image of the Sun on to a sketching-board, the diameter of the image being $10\frac{1}{2}$ inches. It is only necessary to rule one horizontal diameter of the circle prepared to receive the solar image in order to adjust the drawing to the apparent north point. This is effected by causing a spot to run truly along this diameter by means of the slow-motion rods of the telescope. It is better to choose a small spot for this purpose.

As the heliographic latitude of the centre of the Sun's disc varies between $\pm 7^\circ\ 15'$, eight orthographic projections of the meridians and parallels, corresponding to the fifteen positions of the pole ± 0 to $\pm 7^\circ$, were drawn with great care and exactness by Mr. William McKeon. They were then traced on glazed linen, so as to permit of their being placed over a drawing for measurement, all the details on the drawings being easily seen through the projection. Moreover they are very durable, and, as experience has proved, not liable to shrink, thus being an excellent substitute for costly projections ruled on glass.

Each of the projections has, moreover, a scale of degrees from 0 to $\pm 30^\circ$, ruled along its limbs north and south of the projection of the Sun's equator, to enable the disc to be adjusted for the varying position of the angle of inclination of the Sun's polar axis, this variation being within the limits $\pm 26^\circ\!\cdot\!5$ from the N. point. With these discs the "Ephemeris for Physical Observations of the Sun," published yearly in the *Companion to the Observatory*, must be used, values of P, D, and L being easily interpolated for the day and time of observation.

The use of the projections will be best illustrated by an example. To find the heliographic coordinates of a sun-spot drawn in the S.F. quadrant of the Sun at $9^{h}\ 37^{m}$ G.M.T. on 1893 Aug. 9. For this date and time $P = +14^\circ\!\cdot\!33$, $D = +6^\circ\!\cdot\!46$, $L = 266^\circ\!\cdot\!56$. Measure the apparent latitude and longitude by means of the projection for $D = \pm 6^\circ$, adjusting it with reference to the horizontal diameter ruled on the picture to the reading

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+14°·3. The readings are for latitude −16°·8, for longitude −53°·5. If the spot should be large, take the readings of its edges and deduce the mean. Since the meridians and parallels are drawn at intervals of 10°, in estimating degrees and parts of a degree it will be necessary to use subsidiary scales, allowing for the gradual foreshortening as the limbs are approached. These can be ruled on slips of paper as required.

The projection readings must now be corrected. The latitude of the centre of the disc at the time of observation was +6°·46, and we have used the projection corresponding to latitude +6°. If the spot had been situated on the central meridian it is evident that its latitude being south the correction to be applied would have been +0°·46. But it is distant 53°·5 from the central meridian. The distance between the projections of the equators on two successive discs is evidently 1° on the central meridian and 0° at the points to which they converge on or in the immediate neighbourhood of the limb. At intermediate meridians it is successively and proportionally less than 1°, reckoning from the central meridian. The factor to be applied is the natural cosine of its angular distance from the central meridian, and the correction to the apparent latitude depends on the apparent longitude. In applying this correction it is only necessary to employ the first two figures of the value of the natural cosine. In the present instance, therefore, +·46 × ·59 is the correction to be applied. Hence the true latitude of the spot is −16°·5 S. We stated that L for the date and time of the drawing was 266°·56. This value has been thus derived: At noon on August 9 L for the central meridian was 265°·25, but the outline of the spot was traced at $9^{h}\ 37^{m}$, or $2^{h}\ 23^{m}$ before noon; and although the difference in longitude of the central meridian for an interval of twenty-four hours varies with the time of the year, yet it is sufficiently accurate to reckon the mean difference in longitude for this period as 13°·22. Hence the differences for hours and minutes can easily be deduced. For $2^{h}\ 23^{m}$ the correction to be applied is +1°·31. Hence L at the time of the drawing was 266°·56, and the true longitude of the spot becomes 266°·56 −53°·5=213°·1. The Greenwich results for the same spot are lat. −16°·1, long. 213°·1. As the projections are made for every whole degree, the error, either in the parallels or the meridians of any projection used, ought not to exceed what would correspond to a difference of half a degree of polar tilt, since each whole degree projection is used only for dates corresponding to the nearest half degree at either side of it. This error is reduced by the use of the table of natural cosines as explained. The error of the meridians due to a difference of half a degree of tilt is in the spot zones about two-tenths of a degree as a maximum. This error also can be eliminated by measuring the longitude a second time by means of a projection corresponding to half a degree polar tilt in the reverse sense and taking the mean of the two readings. In this manner the error in longitude can be reduced to about 0°·1.

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As pointed out in the "Researches on Solar Physics," by W. De la Rue, B. Stewart, and B. Lœwy (*Phil. Trans.* 1869, p. 10), the measured distance of a spot from the centre of a drawing is greater than its true angular distance from the central meridian of the Sun. In using orthographic projections for deducing the positions of the sun-spots an error due to this difference will always enter into the measurements. But if a spot is followed across the disc the errors on both sides of the central meridian, being of opposite sign, will neutralise each other in determining the mean longitude of the spot. But this is not the case for the latitudes, which ought therefore to be always greater than the true latitude. The maximum value of this error is between 0°·2 and 0°·3 for projections of 10½ inches diameter. Hence in using projections such as those described the longitudes should be much nearer the Greenwich standard positions than the latitudes, and for the latter the maximum cumulative error from all causes ought not to exceed 0°·4. Having at hand 82 positions of sun-spots for the year 1889, originally measured for a different object, we have compared them with the Greenwich positions for the same spots on the same dates. Of these measures 5 of the longitude and 9 of the latitude have been rejected as anomalous, either because the spots were very near the limb where measures are made with difficulty, or because they were changing their positions rapidly. Of the remaining 77 positions in longitude 3 differ 0°·6, 6 differ 0°·5, and 5 differ 0°·4 from the Greenwich positions, the remaining 63 varying between the limits 0° to 0°·3. Of the 73 positions in latitude used in the comparison 5 differ 0°·7, 5 differ 0°·6, 4 differ 0°·5, and 4 differ 0°·4 from the Greenwich positions, while 55 are within the 0°·3 limit. If, however, we take the mean positions of the groups the accordance is much closer, as the following table shows. For the omitted groups from the table either we have no positions from the drawings, or in two cases, groups 2099 and 2100, the results are vitiated by the anomalous positions alluded to above.

No. of Group.	Greenwich Mean.		Stonyhurst Mean.		Difference G—S.		No. of Comp.
	Long.	Lat.	Long.	Lat.	Long.	Lat.	
2083 (*a*)	318°·64	− 7°·89	318°·54	− 8°·11	+0°·10	−0°·22	8
2085	311·53	+ 5·88	311·53	+ 5·75	0	+0·13	4
2086	315·10	+ 5·10	315·20	+ 4·72	−0·10	+0·38	5
2088	217·43	− 1·55	217·43	− 1·83	0	−0·28	4
2090 (*a*)	31·10	− 5·42	31·08	− 5·60	+0·02	−0·18	11
2092	36·31	− 7·91	36·30	− 8·32	+0·01	−0·41	9
2094 (*a*)	343·55	−23·40	343·65	−23·45	−0·10	−0·05	2
2097 (*a*)	163·78	−20·35	163·88	−20·58	−0·10	−0·23	6
2097 (*b*)	155·43	−21·60	155·53	−21·65	−0·10	−0·05	4
2103	151·85	−20·65	151·73	−21·12	+0·12	−0·47	6
2110	101·13	−26·53	101·03	−26·63	+0·10	−0·10	3

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It is to be understood, of course, that the drawings can also be measured and reduced according to the methods detailed in Carington's researches on solar physics, our present object being to show how closely the simple method of projection measurement approximates to those that are more accurate. The appended verification of the exactness with which the projections have been drawn, and a method of measurement deduced from them which is independent of their use, is due to the Rev. James Cullen, of this college.

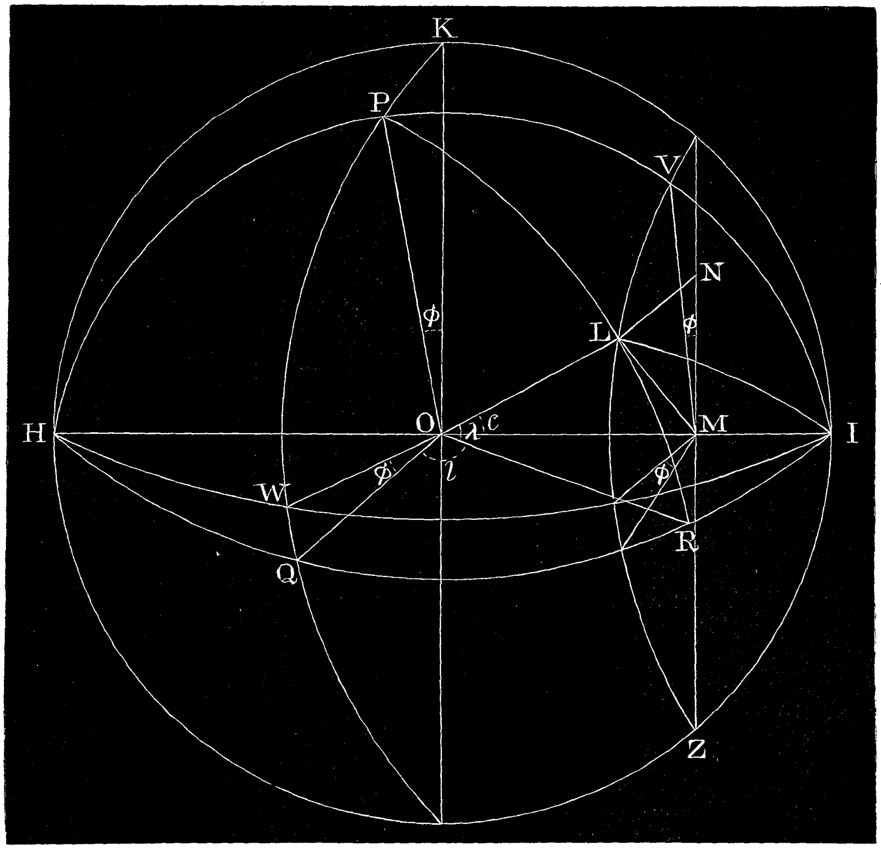

Fig. 1.

1. The discs have been verified in the following manner :—In fig. 1 let P be the pole of the Sun, HQRI his equator, HWI the GC of which K is the pole. If, then, L is a spot, its coordinates (heliographic) are QR and RL. It is clear that as P moves in the meridian KPWQ, L will move in the small circle VLZ, the radius of which is LM=OL sin $c=r$ sin c (r=radius of sphere). But c=LI, the hypotenuse of the right-angled triangle LIR ; LR =λ (the latitude) ; RI=90 − l (l being the longitude).

$$\therefore \cos c = \cos \lambda \sin l.$$

Hence

$$LM = r\,(1 - \cos^2 \lambda \sin^2 l)^{\frac{1}{2}},$$

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and if $\angle$ LIV $=\theta$

$$\mathrm{MN}=\mathrm{LM}\cos(\theta+\phi).$$

Now $\theta=\mathrm{I}$, the angle of the triangle PLI; $\mathrm{PL}=90^\circ-\lambda$; $\mathrm{PI}=90^\circ$; and

$$\mathrm{LI}=c=\cos^{-1}\{\cos\lambda\sin l\};$$

whence applying the formula

$$\cos \mathrm{A}=(\cos a-\cos b\cos c)/\sin b\sin c,$$

where $\mathrm{A}=\theta$, $a=\mathrm{PL}$, $b=90^\circ$, we get

$$\cos\theta=\sin\lambda/(1-\cos^2\lambda\sin^2 l)^{\frac{1}{2}}.$$

Therefore substituting for θ in $\mathrm{MN}=\mathrm{LM}\cos(\theta+\phi)$, we have

$$\mathrm{MN}=y=r(\cos\phi\sin l-\sin\phi\cos\lambda\cos l)\quad\ldots\ldots\quad(1)$$

$$\mathrm{OM}=x=r\cos c=r\cos\lambda\sin l\quad\ldots\ldots\ldots\quad(2)$$

The curves giving the meridians and parallels of latitude are found by making $l=\alpha$ (a constant) and eliminating λ between (1) and (2) or making $\lambda=\beta$ and eliminating l.

Knowing ϕ, λ, and l, we can find the ordinates and abscissæ of any of the points of intersection of the parallels of latitude and meridians of longitude on the discs. This has been applied to the Stonyhurst projections, and they were found to be absolutely correct.

2. As stated in the paper, a proportional table was constructed empirically from measurements taken on the discs in order to apply a correction to the latitude due to the apparent longitude of the spot. That this is a table of natural cosines can be thus shown.

For a projection ϕ_1, say, we have from (1)

$$y_1=r(\cos\phi_1\sin\lambda-\sin\phi_1\cos\lambda\cos l),$$

and similarly for ϕ_2 another projection

$$y_2=r(\cos\phi_2\sin\lambda-\sin\phi_2\cos\lambda\cos l)$$

$$\therefore y_1-y_2=r\{(\cos\phi_1-\cos\phi_2)\sin\lambda-(\sin\phi_1-\sin\phi_2)\cos\lambda\cos l\}$$

and as ϕ_1 and ϕ_2 always lie between 0° and 8°, $\cos\phi_1$ and $\cos\phi_2$ lie between 1 and ·990; so that $\cos\phi_1-\cos\phi_2$ never can be greater than ·01, and as two successive projections are generally used, it really becomes $\cos\phi^\circ-\cos(\phi+1)^\circ=\cdot 001$ (nearly); and as this is multiplied by $\sin\lambda$ (λ being small for spots), we may neglect the first term in the formula. Hence we have

$$y_1-y_2=r(\sin\phi_2-\sin\phi_1)\cos\lambda\cos l,$$

which varies as $\cos l$ as in the case of equators, for when $\lambda=0$

$$y_1-y_2=r(\sin\phi_2-\sin\phi_1)\cos l$$

by the general formulæ.

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Between 0° and 64° on a disc whose radius is 133·35 mm. (or $5\frac{1}{4}$ in.) an error of 0·1 mm. in the equator cannot cause a greater error than 6′ or 0°·1 in the longitude. But at the limb *one degree* is represented by 0·02 mm.

3. From the consideration of the fundamental equations (1) and (2) a method of measurement of the position of sun-spots can be derived directly from the drawings without the use of any discs. From the *Companion to the Observatory* the position-angle of the north pole of the Sun, &c., can be derived for any day. Let NWSE (fig. 2) represent the picture, draw the polar and equatorial diameters NS and WE. Then from (1) and (2) we get

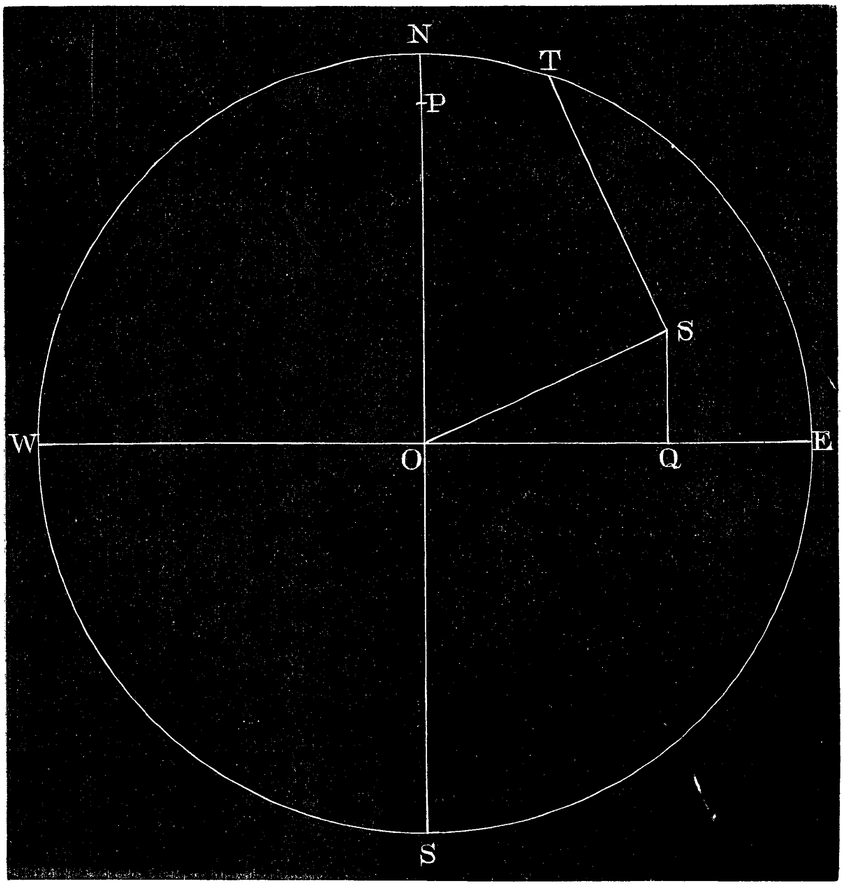

Fig. 2.

$$\cos l = (r^2 \cos^2 \lambda - x^2)^{\frac{1}{2}} / r \cos \lambda,$$

and

$$(y - r \cos \phi \sin \lambda)^2 = \sin^2 \phi \, (r^2 \cos^2 \lambda - x^2)$$

$$\therefore\ y^2 - 2ry \cos \phi \sin \lambda + r^2 \cos^2 \phi \sin^2 \lambda - r^2 \sin^2 \phi \cos^2 \lambda + x^2 \sin^2 \phi = 0\,;$$

i.e.

$$r^2 \sin^2 \lambda - 2ry \cos \phi \sin \lambda + x^2 \sin^2 \phi - r^2 \sin^2 \phi^2 + y^2 = 0,$$

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or

$$(r \sin \lambda - y \cos \phi)^2 = \sin^2 \phi \, \{r^2 - (x^2 + y^2)\}$$

$$\therefore \sin \lambda = [y \cos \phi + \sin \phi \, \{r^2 - (x^2 + y^2)\}^{\frac{1}{2}}]/r.$$

The positive sign must be taken in extracting the square root, since from (1)

$$\sin \lambda > y \cos \phi / r$$

or

$$> \cos^2 \phi \sin \lambda - \sin \phi \cos \phi \cos \lambda \cos l$$

if λ, ϕ, and l are positive.

In practice the following method will facilitate the evaluation of λ. Let S be the spot; join O and S, draw S T perpendicular to O S; join O and T, and draw S Q perpendicular to O E. Then $OQ = x$ and $SQ = y$; $OT = r$. We have therefore

$$ST^2 = OT^2 - OS^2 = r^2 - (x^2 + y^2).$$

Since

$$OS^2 = x^2 + y^2 \therefore ST = \{r^2 - (x^2 + y^2)\}^{\frac{1}{2}}$$

Hence

$$\sin \lambda = (y \cos \phi + ST \sin \phi)/r \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

and

$$\sin l = x/r \cos \lambda \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

λ being found from (3).

We may write

$$\sin \lambda = y \,.\, \alpha + ST \,.\, \beta,$$

where $\alpha = \cos \phi / r$, and $\beta = \sin \phi / r$, these quantities being determined each day: y is negative if measured southwards, β is negative when ϕ is negative, and λ is S. lat. when $\sin \lambda$ is negative.

A convenient way of making use of this method would be to have a right-angled elbow-ruler graduated along both edges in inches or millimetres. Let the zero point be at the elbow; the reading to the centre of the spot gives the x and the reading at right angles to the centre of the disc, the y. Now place the zero point at the centre of the spot, with one edge passing through the centre of the picture; the reading where the other edge cuts the circle gives us ST, and hence the heliographic coordinates can be easily calculated from equations (3) and (4).

4. In the cases where the correction for refraction is needed a modification of the formulæ given in the Introduction to the Greenwich Spectroscopic and Photographic Results can be employed.

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Mercury. By Percival Lowell.

(*Communicated by the Secretaries.*)

The markings on the planet's disc proved at once to be conspicuous. To reduce them to a scale I may say that they were easier to see than all the markings on *Mars* except the most salient ones, such as the so-called Seas. So visible were they at all times—for the air had to be distinctly bad to obliterate them—that observations had not been made more than a day or two, before the rotation period of the planet was patent.

Further communication upon this planet I intend to give later.

Lowell Observatory:
1896 *October* 21.

[In addition to those reproduced, Mr. Lowell sends several prints of drawings of *Mercury*, which have been placed in the Library.—Eds. *M.N.*]

Determination of the Rotation Period and Surface Character of the Planet Venus. By Percival Lowell.

(*Communicated by the Secretaries.*)

The rotation period of *Venus* has hitherto been a matter of doubt. I beg therefore to communicate to the Society the following observations made by me during August, September, and October of this year, which prove that rotation to be performed in the same time as the planet's orbital revolution. I send drawings by me and by my assistant, Mr. Drew. Comparison of the drawings will show their conclusive character, and from them I have been able to construct the accompanying chart.

In the construction of the chart I have taken for the zero longitude that longitude which is upon the central meridian as observed from the Sun when the planet is in the line of apsides of its orbit, either at perihelion or at aphelion, such being the most natural origin to adopt.

In orienting the different drawings of the disc the libration in longitude due to the excentricity of the planet's orbit is added to or subtracted from the value resulting from the planet's orbital revolution. In the case of *Venus* the resulting change is very small, never amounting to more than 47′ at its maximum.

The planet's axis of rotation is substantially perpendicular to the plane of its orbit, as the observations have not yet disclosed any deviation from this position.

In the names for the markings I have selected such as seemed appropriate to the Planet of Love. The markings themselves are